Enhancing Linear System Theory Curriculum with an Inverted Pendulum Robot

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Topics

- Motivation
- LST Course Structure
- MinSeg Robot
- Lessons Learned
- Conclusion
Why enhance LST?

• LST of today similar to that of 50 years ago.

• Applications, though, are so much broader:
  – Driverless cars
  – Financial models
  – Electric grid
  – Drones

• Provide a platform and environment that lets students apply and extend theory.
LST Structure

• Lecture component includes:
  – Block diagrams, modelling, and system properties
  – Causal and LTI systems
  – State space and transfer function relationships
  – Lyapunov and BIBO stability
  – Controllability and observability
  – Closed loop control, estimators, and observers

• Class (PMP) was enhanced by adding:
  – Matlab/Simulink projects to practice the theory
  – Work on the robot to apply control theory to a specific platform
  – Balancing of robot was bonus for the course.
MinSeg model development

Improved engagement:

- Parameters can be hard to measure.
- This naturally leads to questions about accuracy, uncertainty, and how the parameters impact controller performance.
MinSeg Model

inverted pendulum nomenclature

- $P$: Pendulum reference point
- $\alpha$: Angle between MinSeg pendulum and true vertical axis
- $L$: Distance from wheel center to $P$
- $m_p$: Pendulum mass
- $I_p$: Moment of inertia at $P$
- $x$: Trailing distance of wheel
- $r$: Center of wheel
- $m_w$: Mass of wheel
- $r_w$: Radius of wheel
- $I_{m,w}$: Moment of inertia at center of mass of wheel

motor nomenclature

- $T_m$: DC motor torque
- $r$: Motor resistance
- $k_t$: Motor torque constant
- $k_e$: Motor back-EMF constant

mass * acceleration = force

\[
(l_p + L^2m_p)\ddot{\alpha} + L \cos(\alpha) m_p \ddot{x} = -\frac{k_t}{R} \dot{V} + g L \sin(\alpha) m_p + \frac{k_b k_t}{R} \dot{x} - \frac{k_b k_t}{R} \ddot{\alpha}
\]

\[
L \cos(\alpha) m_p r_w \ddot{\alpha} + \left( \frac{l_{cm,w}}{r_w} + m_p r_w + m_w r_w \right) \ddot{x} = \frac{k_t}{R} \dot{V} - \frac{k_b k_t}{R} \dot{x} + \frac{k_b k_t}{R} \ddot{\alpha}
\]

state-space

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-62.1 & -44.6 & 0 & -212 \\
0 & 0 & 1 & 0 \\
6.10 & -10.2 & 0 & -485
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 \\
-90.0 \\
0 \\
0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]
Model Behavior

• Eigenvalues (Unstable):

\[ \text{rank}(E) = \text{rank} \begin{bmatrix} 0.00 & -90.0 & 47700 & -2.53 \times 10^7 \\ -90.0 & 47700 & -2.53 \times 10^7 & 1.34 \times 10^{10} \\ 0.00 & 20.6 & -10900 & 5.77 \times 10^6 \\ 20.6 & -10900 & -5.77 \times 10^6 & -3.06 \times 10^9 \end{bmatrix} = 4, \text{Full Row Rank} \]

• Controllability (System is controllable):

\[ \text{rank}(O) = \text{rank} \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ \ldots & \ldots & \ldots & \ldots \\ 1.90 \times 10^6 & 2.86 \times 10^6 & 0.00 & -1.36 \times 10^9 \end{bmatrix} = 4, \text{Full Row Rank} \]

• Observability (System is observable).
LQR Feedback Controller

• An LQR (Linear-Quadratic Regulator) controller applies gains to all the state variables:

\[ \dot{x} = Ax + Bu \]
\[ \dot{y} = Cx + Du \]
MinSeg with LQR impulse response

LQR Feedback Controller

$$K_{\text{Closed Loop, Actual}} = [-93.0864 \quad -18.0634 \quad 9.93776 \quad 52.6578]$$

![Graph showing impulse response with markers and lines representing different variables.](image)
Feedback Controller Development

P Controller

• Begin with proportional, “P” feedback. Block diagram and modified transfer function:

\[ \hat{x} = Ax + Bu \]
\[ y = Cx + Du \]

• With Matlab a gain of \(-67.4 \, \text{Volt/rad}\) is found.
P Controller Response

- Response of the model and the MinSeg robot:
PI Feedback Controller

• Add integral, “I” control:

• Gain of $K_p = -1740 \, \frac{\text{Volt}}{\text{rad}}$ and $K_I = -8070 \, \frac{\text{Volt}}{\text{rad}}$.

• This is a problem: Excessive control effort.
PI controller impulse response

- Lowered required response time and found more realistic gains:

![Pendulum angle, α](image)

**Slow ripple**
About those wheels...

PID Feedback Controller

• Added differential gain, $k_d$, to eliminate ripple and wheels need controller:

$$\dot{x} = Ax + Bu$$
$$\dot{y} = Cx + Du$$
PID + P controller impulse response

- Found $k_p$ and $k_i$, adjusted $k_d$, and added $k_w$ gain to achieve desired response.
Summary

• Class
  – Enhanced engagement
  – Robot improved understanding of concepts (pole placement, observers)

• Lab
  – Expanded learning (complimentary filter design)
  – Introduced additional depth to control theory (Motor controls and ultrasonic distance sensor)
Conclusion

• Successful experiment!

• Next steps:
  – Explore Kalman filtering
  – Apply lab structure to other topics
Questions?

-URL goes here-